Sequencing JIT Mixed Model Assembly Lines Under Station-Load and Part-Usage Constraints using Lagrangean Relaxations

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1. Introduction

Many production systems make use of assembly lines for mass production. An assembly line is made up of several workstations through which the work in progress on a product flows. The stations are linked together by a transport system, like a conveyor belt, whose mission is to supply materials to their main flow and to move the production items from one workstation to the next. In ideal circumstances, an assembly line is responsible of the production of a single product. In this context, manufacturing operations required by the product are divided into a set of tasks and each task is assigned to a single workstation. The set of tasks assigned to one workstation is known as workload, and the workers and robots assigned to each workstation have a constant time to develop the tasks. The highest workload defines the cycle time, and it determines the production rate of the line.

On the contrary, a mixed model assembly line assembles different units, but the similitude between each unit is high enough to consider each unit as identical during the assembly line design. Mixed model lines are very frequent in the automotive industry, where diversity of options inside a single product are a strong demand from the customers. Practitioners are then faced with two different problems. The first one, the balancing of the assembly line, is equivalent to the single model case, but mean task operation times are used to balance the line; the second one consists in sequencing the line in accordance to the differences between the units.

The latter problem has been tackled in the literature by proposing procedures to sequence the units in the line. In one hand, the initiatives from the Artificial Intelligence field have been interested in the feasibility of the final sequence according to workload constraints in one or several workstations. On the other hand, approaches from the Operations Research and Operations Management fields have been focused in the regularization of parts usage via the minimization of an objective function measuring the difference between the real sequence and an ideal one.

The present work is focused on the formulation and resolution of an approach combining both objectives, and extends a previous work found in [3]. The preliminary results from the new proposed procedure, based on the Lagrangean relaxation of the mathematical model of the problem are very promising, improving the previously reported results from [3] and solving to optimality 32 of the 36 instances found in the literature.

The rest of the work is structured as follows. In section 2 a brief literature review is presented. Section 3 is devoted to the proposed procedure to solve the problem, while section 4 exposes the results of the procedure comparing them with those previously reported. Finally, in section 5 the conclusions of the present work are put forward and several extensions are hinted.

2. State of the Art

The literature collects two main approaches to the sequencing of mixed model assembly lines, regarding the desired conditions of the sequence.

The first one can be traced back to the JIT production system in TOYOTA as documented by Monden, see [9]. The proposed objective is to find a sequence with option and consumption rates as regular as possible. Regularity is measured via the evaluation of the differences between mean consumption rates and the ideal rates. This proposal tries to blend the station loads of workstations, to establish a constant flow in the supply chain, and thus, to minimize stock on hand, space requirements on the assembly lines and other operative costs.

Several authors, see [10], presented the second one as is usually known as the Car Sequencing Problem (CSP). In this approach the objective is based on the fulfillment of several constraints required for proper operation. The approach is based on the assumption that different options required by the units affect station loads, and station loads must be taken into account. When multiple units requiring an option appear consecutively in a sequence, the workstations in charge of the required option may not be able to end their job in the allotted time, generating a situation that, in the best case, additional workforce can solve, but incurring in additional costs. To obtain sequences where this situation does not occur, the solution is forced to fulfill several constraints per option requiring that a given number of units with this option do not appear in a given number of consecutive positions of the sequence.

In [3], a mixed model is proposed and solved, combining previous works from both approaches:

a) The regularization of the sequence is based on production rates, see [6]. The proposal associates regularity with a discrepancy function between ideal due dates, related to ideal positions in the sequence, and real due dates, related to the real position in the sequence. Under the due dates given by the authors, the problem can be easily solved by the Earliest Due Date (EDD) rule. In the proposed procedure, the problem will be solved as an Assignment Problem as indicated in [7].

b) The car sequencing constraints as given in [2] are imposed as additional constraints to the model. The constraints are given in the form, maximum H_o units requiring option o, in every N_o consecutive positions in the sequence.

The original proposal to tackle the problem is to use a column generation approach to solve the mathematical model. The authors report results for a specially tailored dataset, due to the lack of previous references, indicating that the problem is capable of solving instances with up to 40 units and 3 different options to optimality, as well as providing lower and, sometimes, upper bounds to the optimal solution.

3. Solving the problem using Lagrangean Relaxations

Another possible approach to solve the mathematical model is to use a Lagrangean relaxation approach, [5]. The method is not only capable of giving lower bounds to the solution of the instances of the problem, but also to generate upper bounds during the bounding phase. After the bounding phase, the generated problem can also be used to generate new bounds to combine with an enumerative procedure like a Branch and Bound one.

Lagrangean relaxation approaches are based on relaxing certain constraints, usually the hard constraints of the problem, and insert them in the objective function with a penalizing factor. The new problem has an additional set of variables, known as Lagrange multipliers. When the Lagrange multipliers adopt a given value, the remaining problem can be solved efficiently. Obviously a new problem appears, associated to find the best possible set of Lagrange multipliers.

In the present work, the relaxed constraints are those originated by the CSP, and the relaxed problem is solved as an Assignment Problem (AP). The weight of the multipliers is obtained by a subgradient method [5], and the solution of each related AP instance is verified for feasibility. If the subgradient method is not sufficient to assess optimality, the procedure tries to solve the instance via Branch and Bound, using a modified Assignment Problem to calculate bounds for every partial solution.

4. Computational Experience

To assess the quality of the proposed procedure, the algorithm was programmed in C. The instances from [3] were used and the results provided by the presented procedure and their reported results are compared. Let us note that the instances have three special characteristics: (1) all instances have a solution, (2) all instances have maximum work load compared to the maximum possible work load given by the constraints and (3) the dataset consists in 36 instances with a number of units between 10 and 50, and a number of options between 3 and 7.

The Lagrangean relaxation procedure is able to obtain the optimal solution, and verify its optimality, in 19 of the 36 instances, in comparison to the 14 instances solved to optimality in [3]. When the Branch and Bound procedure is used, with a time limit of 15 minutes per instance, the procedure is capable of solving 32 instances with running times ranging from 0 to 700 seconds and a mean running time of 22 seconds using a 1,8 Ghz. Pentium IV computer.

5. Conclusions and Extensions

The proposed procedure seems to be comparable to the newest procedures found in the literature, [4]. As indicated in [3] it can also be used to generate cyclic sequences, very often used in practice. Further experiments with larger instances, with very different characteristics as those adapted from the CSPLIB (www.csplib.org), have failed to provide good solutions. To tackle bigger instances a different approach, as a metaheuristic, using the same ideas, is being developed.

Finally, it is important to cite that the current procedure can be used with different objective functions, as those given in [8]. Further work should be required to adapt the procedures to the proposals of [1] or [9], whose formulation resembles more clearly the desires of the automotive industry.

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