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Solving the *ORVP* with Preservation of the Production Mix using *BDP*

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Abstract We present a sequencing problem given on *JIT* (Just In Time) manufacturing environments, with the objective of minimizing the variation of production rates (*ORV*: Output Rate Variation). We propose an extension of this problem based on to require, to the sequences, the preservation of the production mix throughout the manufacturing of the products. To solve the *ORVP* (Output Rate Variation Problem) and the extended problem, we propose algorithms based on *BDP* (Bounded Dynamic Programming).

Keywords: Sequencing, Just In Time, Bounded Dynamic Programming.

1.1 Introduction

Circulating units in automobile production and assembly lines are not always identical; some units have a certain degree of similarity (families), but they may vary in the use of resources in workstations and the consumption of components.

At each workstation, we can talk about the use of human resources, automated systems and tools to which is assigned an average workload, measured in units of time, with a maximum average value called cycle.

In addition, each product is created with parts (components), according to the *BOM* (Bill of Materials), which are incorporated into the *WIP* (Work In Progress) following the flow of the line to obtain the final products.

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These manufacturing lines (mixed-products) are very common in *JIT* and *DS* (Douki Seisan) environments and allow to manufacture, using the same production system, variants of one or more products.

This desirable flexibility determines the order or sequence in which the units on the line should be handled to satisfy three general principles: (I) a drastic reduction in component stock and semi-manufactured products, (II) efficient use of the avalaible manufacturing time and (III) reduction of the work overload to the minimum. Boysen et al. (2009) established three sequencing problems types in these manufacturing environments: (1) Mixed-model sequencing, (2) Car sequencing problem, and (3) Level scheduling.

This paper falls under the principle I and the problem type 3. Specifically, we focus on the study of the *ORVP* (Output Rate Variation Problems) and *PRVP* (Product Rate Variation Problems) and we propose several approaches to treat both problems at once. To solve the selected alternative in this paper we use a procedure based on Dynamic Programming using bounds (Bautista et al., 1996).

1.2 *ORVP* and *PRVP*

1.2.1 The ORVP

The *ORVP* is described for the first time in a work by Monden (1983) dedicated to the Toyota production system, and its name comes from Kubiak (1993).

The problem is to sequence, regularly, a total of D products, grouped into a set I of product types, of which d_i are of type i ($i = 1,\ldots,|I|$). The components are grouped into a set J. A product unit of type i requires $n_{j,i}$ units of component type j ($j = 1,\ldots,|J|$). The problem objective is to minimize the variation in consumption rates of the components during the manufacturing of products.

The ideal consumption rate (constant over time) of component j is:

$$\dot{n}_j = \frac{1}{D}\sum_{i=1}^{|I|} n_{j,i} \cdot d_i \qquad j = 1,..,|J| \tag{1.1}$$

Therefore, the ideal consumption of the component j when t products were manufactured must be:

$$Y^*_{j,t} = t \cdot \dot{n}_j \qquad j = 1,..,|J|;\ t = 1,..,D \tag{1.2}$$

Moreover, when t products were manufactured, of which $X_{i,t}$ are of type i ($i = 1,\ldots,|I|$), the actual consumption of the component j ($j = 1,\ldots,|J|$) is:

$$Y_{j,t} = \sum_{i=1}^{|I|} n_{j,i} \cdot X_{i,t} \qquad j = 1,..,|J|;\ t = 1,..,D \tag{1.3}$$

The discrepancy or distance between the actual and ideal consumption of the component j when have passed through the line t product units is:

$$\delta_{j,t}(Y) = Y_{j,t} - Y^*_{j,t} \qquad j = 1,..,|J|;\ t = 1,..,D \tag{1.4}$$

Under these conditions, non-regular consumption of components for D products can be measured through the discrepancies defined in (1.4); that is:

$$\Delta_R(Y) = \sum_{t=1}^{D}\sum_{j=1}^{|J|}\left|\delta_{j,t}(Y)\right|, \quad \Delta_E(Y) = \sum_{t=1}^{D}\sqrt{\sum_{j=1}^{|J|}\delta^2_{j,t}(Y)}, \quad \Delta_Q(Y) = \sum_{t=1}^{D}\sum_{j=1}^{|J|}\delta^2_{j,t}(Y) \tag{1.5}$$

Where $\Delta_R(Y)$, $\Delta_E(Y)$ and $\Delta_Q(Y)$, are respectively the global rectangular, Euclidean and quadratic discrepancies of the consumption of components.

Let be the set of functions $\Im_Y = \{\Delta_R(Y), \Delta_E(Y), \Delta_Q(Y)\}$, then the resulting single-objective models for the *ORVP* are:

M_ORV Models:

$$Min\ f\ \left(f \in \Im_Y\right) \tag{1.6}$$

Subject to:

$$\sum_{t=1}^{D} x_{i,t} = d_i \qquad i = 1,..,|I| \tag{1.7}$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \qquad t = 1,..,D \tag{1.8}$$

$$x_{i,t} \in \{0,1\} \qquad i = 1,..,|I|\ ;\ t = 1,..,D \tag{1.9}$$

The variables $x_{i,t}$ ($i = 1,\ldots,|I|, t = 1,..,D$), subject to constraints (1.9), are binary variables that take the value 1 if a product unit of type i occupies the t^{th} position of the sequence and 0 otherwise; constraints (1.7) impose the demand satisfaction of all products; and constraints (1.8) indicate that only one product unit can be assigned to at each position in the sequence. Obviously, the link between the variables $x_{i,t}$ and $X_{i,t}$ is:

$$X_{i,t} = \sum\nolimits_{\tau=1}^{t} x_{i,\tau} \qquad i = 1,..,|I|\ ;\ t = 1,..,D \tag{1.10}$$

To solve the problem various heuristics (Monden, 1983; Bautista et al., 1996; Jin and Wu, 2002) and exact procedures (Bautista et al., 1996; Miltenburg, 2007) have been proposed.

1.2.2 The PRVP

The *PRVP* is described for the first time in a work by Miltenburg (1989) and its name comes from Kubiak (1993).

The problem is to sequence, regularly, a total of D products, grouped into a set I of product types, of which d_i are of type i ($i = 1,\ldots,|I|$) so that the production rates are maintained as constant as possible along the time in that the products are manufactured.

The *PRVP* is a specific case of *ORVP* if we impose: (1) a bijective application between the sets I and J (therefore $|I|=|J|$) and (2) each product type requires one unit of component related through these application.

In this case, we can define the following objective functions of non-regularity in production (X) between the actual and ideal productions over time:

$$\Delta_R(X)=\sum_{t=1}^{D}\sum_{i=1}^{|I|}\left|\delta_{i,t}(X)\right|, \quad \Delta_E(X)=\sum_{t=1}^{D}\sqrt{\sum_{i=1}^{|I|}\delta_{i,t}^2(X)}, \quad \Delta_Q(X)=\sum_{t=1}^{D}\sum_{i=1}^{|I|}\delta_{i,t}^2(X) \tag{1.11}$$

Where:

$$\delta_{i,t}(X)=X_{i,t}-X_{i,t}^{*}=\sum\nolimits_{\tau=1}^{t}x_{i,\tau}-t\frac{d_i}{D} \qquad i=1,..,|I|\ ;\ t=1,..,D \tag{1.12}$$

Let $\delta_{i,t}(X)$ be the discrepancy or distance between the actual and ideal production of product i when t product units were manufactured.

If we define the set of functions $\Im_X=\{\Delta_R(X),\Delta_E(X),\Delta_Q(X)\}$, the resulting single-objective models for the *PRVP* are:

M_PRV Models:

$$Min\ f' \ \left(f'\in\Im_X\right) \tag{1.13}$$

Subject to: (1.7) – (1.9) from *M_ORV*

1.2.3 Relation between ORVP and PRVP

The *PRVP* is a particular case of *ORVP* $(I=J)\wedge(n_{j,i}=\delta_{j,i}:\text{Kronecker delta})$.

On the other hand, to establish a link between the solutions of both problems, we will build on the properties derived from preserving a production mix when manufacturing products units over time.

Let $X_{i,t}^{*}=t\cdot d_i/D$ be the number of units of product type i ($i=1,\dots,|I|$), of a total of t ($t=1,\dots,D$) units that should ideally be manufactured to maintain the production mix. And, let $\vec{X}^{*}=(X_{1,1}^{*},\dots,X_{|I|,D}^{*})$ be the ideal point of cumulative production.

Then, for the ideal point $\vec{X}^{*}$ the following is fulfilled: $\delta_{i,t}(X)=X_{i,t}-X_{i,t}^{*}=0$ ($i=1,\dots,|I|$, $t=1,\dots,D$); and therefore, $\Delta_R(X)$, $\Delta_E(X)$ and $\Delta_Q(X)$ are optimal and are equal to zero. In addition, the point $\vec{X}^{*}$ has the property of regularizing the consumption of components. In effect:

Theorem 1. For the ideal point $\vec{X}^{*}$, then: $\delta_{j,t}(Y)=Y_{j,t}-Y_{j,t}^{*}=0$ ($i=1,\dots,|I|$, $t=1,\dots,D$).

Proof. In this case, we have:

$$Y_{j,t} = \sum_{i=1}^{|I|} n_{j,i} \cdot X^*_{i,t} \Leftrightarrow Y_{j,t} = \sum_{i=1}^{|I|} n_{j,i} \frac{d_i}{D} t = t\left(\frac{1}{D}\sum_{i=1}^{|I|} n_{j,i} \cdot di\right) = t \cdot \dot{n}_j = Y^*_{j,t}$$

Therefore,

$$\delta_{j,t}(Y) = Y_{j,t} - Y^*_{j,t} = 0 \qquad j = 1,..,|J| \ ; \ t = 1,..,D \tag{1.14}$$

Corollary 1: For point $\vec{X} = \vec{X}^*$, $\Delta_R(Y) = \Delta_E(Y) = \Delta_Q(Y) = 0$. Consequently, the functions of global discrepancies, rectangular, Euclidean and quadratic, of the consumption of components are optimal.

1.3 Models for the *ORVP* with Production Regularity

To address the *ORVP* and *PRVP* at once, we can use at least two ways of working: (1) Address the problems together as a multi-objective problem and to do this, formulate and use new models with bi-objective functions, and (2) add to the original *ORVP* models a set of constraints that guarantee the preservation of production mix throughout the working day.

1.3.1 Bi-objective ORVP and PRVP models

Based on Theorem 1 and the conclusions derived from it, we can state that the preservation of production mix is in line with the regularity of the consumption of components; so, if both properties are desirable, it is reasonable to formulate the following bi-objective models:

M_ORV_PRV Models:

$$(Min\ f) \wedge (Min\ f') \quad (f \in \Im_Y, f' \in \Im_X) \tag{1.15}$$

Subject to: (1.7) – (1.9) from *M_ORV*

1.3.2 ORVP Models with Production Mix Restriction (pmr)

At the conclusion of Theorem 1, also, we can control the regularity of production in sequences, if we limit the values of the variables of cumulative production, $X_{i,t}$ ($i = 1,\ldots,|I|; t = 1,\ldots,D$), which must be whole integers, to be the integers closest to the ideal values $X^*_{i,t} = d_i \cdot t/D$. That is:

$$\left\lfloor \frac{d_i}{D} \cdot t \right\rfloor \le X_{i,t} \le \left\lceil \frac{d_i}{D} \cdot t \right\rceil \qquad i = 1,..,|I| \ ; \ t = 1,..,D \tag{1.16}$$

Where $\lfloor x \rfloor$ and $\lceil x \rceil$ are greatest integer less than or equal to x and smallest integer greater than or equal x, respectively.

In this way, from *M_ORV* reference models, we have:

M_ORV_pmr Models:

$$Min\, f \ \left(f \in \Im_Y\right) \tag{1.17}$$

Subject to: (1.7) – (1.9) from *M_ORV* and (1.16)

In this paper, we use the *M_ORV* and *M_ORV_pmr* models with the function $\Delta_Q(Y)$ as objective function f.

1.4 The Use of the *BDP* to the *ORVP* and *ORVP_pmr*

Bounded Dynamic Programming (*BDP*) is a procedure that combines features of dynamic programming with features of branch and bound algorithms related to the use of overall and partial bounds of the problem. The procedure determines an extreme path in a multistage graph with $D+1$ stages, and explores some or all of the vertices at each stage t ($t=0,\ldots,D$) of the graph and uses overall bounds of the problem to remove, discard and select, stage by stage, the vertices most promising, then develop these, until to reach the last stage D.

To solve the *ORVP*, the algorithm *BDP* and the system of partial and overall bounds, *BOUND4,* designed to this problem (Bautista et al., 1996) are used and the minimization of the function $\Delta_Q(Y)$ is fixed as objective.

Regarding the resolution of the *ORVP_pmr*, the above procedure has been adapted for the *ORVP* adding a mechanism to remove, at each stage ($t=0,\ldots,D$), the vertices that do not satisfy the preservation conditions of production mix (1.16). This elimination rule reduces significantly the search space of solutions, because the number of vertices $H(t)$ to consider at each stage t of the graph is limited by the number of product types $|I|$ as follow:

$$H(0)=H(D)=1; \quad H(t) \le \frac{|I|!}{\lceil |I|/2 \rceil! \cdot \lfloor |I|/2 \rfloor!} \qquad t=1,\ldots,D-1 \tag{1.18}$$

For example, in a set of instances with $|I|=4$ for the *ORVP_pmr*, a maximum window width of $H=6$ will be sifficient to guarantee all optima.

1.5 Computational Experiment

The experimental data correspond to 225 instances of reference (Jin & Wu, 2002) with 45 demand plans into 5 blocks (*B*), and 5 product-component structures (*E*), which represent the *BOM*. All instances have four product types ($|I|=4$) and a total demand of 200 units ($D=200$).

To obtain optimal solutions for the 225 instances from *M_ORV* and *M_ORV_pmr* models, the *BDP* was used, under the following conditions: (1) *BDP* procedure programmed in C++, using gcc v4.2.1, running on an Apple Macintosh iMac computer with an Intel Core i7 2.93 GHz processor and 8 GB RAM using MAC OS X 10.6.7; (2) six windows width (*H*) were used, with values 1, 6, 64, 128, 512 and 1024, to reach the optima, but to demonstrate all of them, eight windows width were necessary (1, 6, 64, 128, 512, 1024, 2048 and 4096). The initial solution, Z_0, for each window width was the solution obtained by *BDP* with the previous window width, except for $H=1$, where Z_0 was established as ∞.

In the computational experiment, given the models for *ORVP* and *ORVP_pmr*, the functions $\Delta_Q(Y)$, $\Delta_Q(X)$ and the set of instances E: (1) we determine the solution with the best value for $\Delta_Q(Y)$ that both models offer for each instance $\varepsilon \in \mathrm{E}$; (2) If those solutions are called $S^*_{ORV}(\varepsilon)$ and $S^*_{ORV\_pmr}(\varepsilon)$, then we measure the relative percentage deviations (*RPD*) for the values of the functions $f \in \left\{\Delta_Q(Y), \Delta_Q(X)\right\}$ as shown in (1.19).

$$RPD(f,\varepsilon) = \frac{f\left(S^*_{ORV}(\varepsilon)\right) - f\left(S^*_{ORV\_pmr}(\varepsilon)\right)}{f\left(S^*_{ORV}(\varepsilon)\right)} \cdot 100 \qquad \left(f \in \left\{\Delta_Q(Y), \Delta_Q(X)\right\};\ \varepsilon \in \mathrm{E}\right) \tag{1.19}$$

In tables 1.1 to 1.4 the main results are collected.

Table 1.1 Minimum, maximum and average CPU times needed to obtain optimal solutions given by models *ORV* and *ORV_pmr*.

	$CPU_{\min}$	$CPU_{\max}$	$\overline{CPU}$
ORV	0.64	80.03	11.32
ORV_pmr	0.17	0.22	0.21

Table 1.2 Number of optimums reached for each window width.

	H=1	*H=6*	*H=64*	*H=128*	*H=512*	*H=1024*
ORV	9	121	174	199	223	225
ORV_pmr	3	225	-	-	-	-

Table 1.3 Number of optimums demonstrated for each window width.

	H=1	*H=6*	*H=64*	*H=128*	*H=512*	*H=1024*	*H=2048*	*H=4096*
ORV	0	0	19	51	177	210	224	225
ORV_pmr	0	225	-	-	-			-

Table 1.4 Average values of *RPD* for the functions $RPD(\Delta_Q(Y))$ and $RPD(\Delta_Q(X))$.

	E1	*E2*	*E3*	*E4*	*E5*	*Average*
$RPD(\Delta_Q(Y))$	-5.02	-5.11	-1.60	-0.06	-10.92	-4.54
$RPD(\Delta_Q(X))$	39.24	19.03	5.11	0.17	37.03	20.11

Table 1.1 shows that *M_ORV_pmr* is fifty times faster than *M_ORV* regarding the average CPU time required to demonstrate the optimal solutions. In addition, the CPU time, spent with *M_ORV_pmr*, does not depend on the instance solved.

In tables 1.2 and 1.3 are shown respectively the optima reached and demonstrated for the window widths used. A window width *H=4096* was necessary to demonstrate the optimums of the 225 instances with *M_ORV* and *H=1024* was sufficient to reach them. For its part, *M_ORV_pmr* reached and demonstrated all the optima with a window width *H=6*.

Finally, regarding the quality of the optima, table 1.4 shows: (1) an average worsening of 4.54% for optimal $\Delta_Q(Y)$ of *M_ORV_pmr* with regard to *M_ORV*; (2) the incorporation of the constraints (1.16) improves by an average of 20.11% the preservation of the production mix ($\Delta_Q(X)$); and (3) more radical average gains in $\Delta_Q(X)$ and average worsenings in $\Delta_Q(Y)$ in those product structures that move away from the possible equivalence between the *ORVP* and the *PRVP*.

1.6 Conclusions

We have presented bi-objective and mono-objective models to the *ORVP* with preservation of the production mix in the *JIT* and *DS* context.

From *M_ORV* and *M_ORV_pmr* models with quadratic function $\Delta_Q(Y)$ for the consumption of components, we have realized a computational experiment with 225 reference instances from the literature using bounded dynamic programming as resolution procedure.

The incorporation of the restrictions to preserve the production mix into *M_ORV*, reduces to one fiftieth the average CPU time with *BDP*, being enough a window width of *H=6* to obtain all the optima in the set of instances employed.

The worsening, in regular consumption of components, by an average of 4.54% of *M_ORV_pmr* over *M_ORV*, is offset by the gain of preservation of production mix of 20.11%.

1.7 References

Bautista J, Companys R, Corominas A (1996) Heuristics and exact algorithms for solving the Monden problem. European Journal of Operational Research, 88/1:101–113.

Boysen N, Fliedner M, Scholl A (2009) Sequencing mixed-model assembly lines: Survey, classification and model critique. European Journal of Operational Research 192/2:349-373.

Jin M, Wu SD (2002) A new heuristic method for mixed model assembly line balancing problem. Computers & Industrial Engineering 44/5:159–169.

Kubiak W (1993) Minimizing variation of production rates in just-in-time systems: A survey. European Journal of Operational Research 66/3:259–271.

Miltenburg J (1989) Scheduling Mixed-Model Assembly Lines for Just-In-Time Production Systems. Management Science, 35/2:192-207.

Miltenburg J (2007) Level schedules for mixed-model JIT production lines: characteristics of the largest instances that can be solved optimally. Internacional Journal of Production Research 45/16:3555-3577.

Monden Y (1983) Toyota Production System. Industrial Engineering and Management Press, Norcross, Georgia.